TOWARDS A MORE ROBUST SPACECRAFT CHARGING ALGORITHM

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Abstract

The NASCAP/GEO computer code, in use worldwide for simulation of spacecraft charging in geosynchronous orbit, is in need of improvement. We discuss the use of the boundary element method (BEM) to allow more flexible geometric specifications while providing more accurate surface electric fields, and the "Method of Patches" to provide more accurate and robust time integration.

Introduction

The NASCAP/GEO (originally NASCAP) code, for three dimensional dynamic simulation of spacecraft charging in the geosynchronous environment, was developed beginning in 1976, with only minor maintenance since about 1984. Since that time, computer processor speed has increased by about a factor of 1000. The costs of computer memory and disk storage have decreased by comparable factors. And, there have been huge advances in software design and mathematical algorithms. Accordingly, the techniques used in NASCAP/GEO to enable cost-effective simulations on the computers of twenty years ago fail to meet the demands and expectations of today's users.

Furthermore, the "kludges" built into NASCAP/GEO to allow calculations to proceed stably in the face of coarse resolution in space and time often produce unsatisfactory or confusing results. Figure 1 shows the results of a calculation that indicates some sudden change occurring at about 800 seconds. In fact, the sudden change was purely internal to the workings of the code, and had little relation to physical reality. Commonly, the progress of charging in time is governed by code internals rather than by physical processes. We have seen sudden (in time) changes caused by coarse spatial resolution as well.

NASCAP/GEO spends most of its time calculating electrostatic potentials in the volume of space surrounding the spacecraft using the Finite Element Method (FEM). However, what is important for charging is the electric field strength at each surface element of the spacecraft. Calculating the volume potential structure is a poor, as well as inefficient, way to get surface electric fields, and this is one source of the problems that "kludges" are in place to overcome. We propose to use the Boundary Element Method (BEM) [Brebbia, 1981] to obtain more accurate surface electric fields, as well as to introduce more flexibility into the spacecraft definition.

Time integration is NASCAP/GEO's second problem. The timescale for overall charging of a spacecraft is about four orders of magnitude faster than the timescale for differential charging. (This wide separation of timescales is often called "stiffness.") NASCAP/GEO uses a first-order implicit timestepping algorthm. However, the treatment is plagued with roundoff problems, leading to ad hoc fixes, and thence to poor representation of timescale. We propose to implement a variation of the "Method of Patches" [Brydon *et al.*,1998], in which the time-scales are separated and then solved exactly within less restrictive approximations than are currently used.

Calculating Potentials and Fields

The original design of NASCAP/GEO called for determining incident currents to a surface via the "reverse trajectory" method. A candidate incident particle is tracked outward from the spacecraft to determine whether it is environmental in origin, and if so to determine its current from the environment model. The Finite Element Method (FEM) was used to calculate potentials in a large volume of external space in order to compute particle trajectories. However, it soon became apparent that, due to large

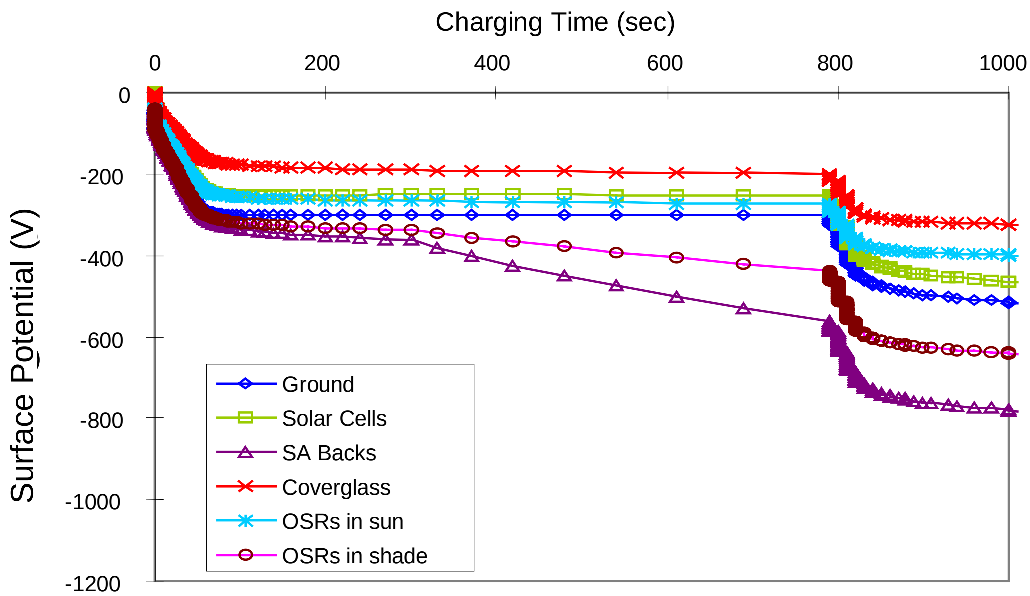

Figure 1. Example of a NASCAP/GEO calculation showing a “glitch” at 800 seconds, as well as generally poor time representation.

sampling errors coupled with vast uncertainty in knowledge of the environment, it was far better to use simple collection models coupled to approximate analytic representations of the environment. (The detailed potential calculations have been used in several applications to determine whether particles incident to detectors are environmental in origin, or originated elsewhere on the spacecraft.)

On the other hand, emission of low energy electrons (photoelectrons and secondary electrons) from surfaces plays a crucial role in charging. For surfaces having a positive (electron-attracting) electric field, these currents are cut off, changing the sign of the surfaces’ net current. Effectively, surfaces for which low energy electron emission is the dominant current satisfy a boundary condition of small, positive electric field.

In NASCAP/GEO, surface electric fields are calculated by numerical differentiation of the space potentials. Not only is this method inaccurate, but it provides little predictive connection between surface potentials and surface fields. This is a major source of problems with NASCAP/GEO calculations. This motivates a switch from the FEM to the Boundary Element Method (BEM), which focuses on the connection between surface potentials and fields. In contrast to the FEM, which solves Poisson’s Equation, $-\varepsilon_0\nabla^2\phi = \rho \approx 0$, the BEM solves for surface potentials using the vacuum Green’s function,

$$(4\pi\varepsilon_0)V_i = \sum_j \int d^2 r_j \frac{\sigma_j}{r_{ij}}$$

which leads to a non-sparse matrix connecting surface potentials and charge densities (equivalent to electric fields). Modern mathematical algorithms are available to solve such a large, non-sparse matrix. Figure 2 [Mandell *et al.*, 1994] shows an example of a BEM calculation (which we developed for another application) of surface magnetic fields in a complex structure.

Advantages of the BEM include:

a) Surfaces are not constrained by gridding, either as to orientation or size.
b) Surfaces need not be compatible, *i.e.*, there is no requirement that one edge be shared by exactly two surfaces.
c) There is no need to zone up the external space.
d) The potential solution, on the surface or in space, is naturally continuous.
e) Accuracy of electric fields is not limited by grid resolution; no numerical differentiation is required.

Variations on the BEM equation provide additional meth-

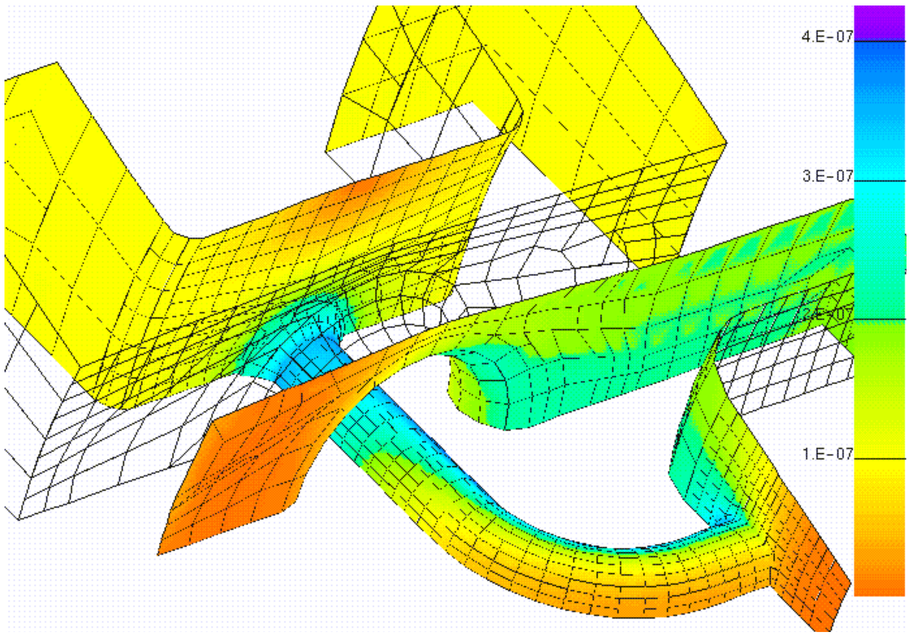

Figure 2. Example of a BEM calculation of surface magnetic fields of a complex structure.

ods for improving the robustness and stability of charging calculations. For example, its gradient directly relates electric fields to surface charges, and its time derivative globally relates potential changes to currents.

Time Integration Algorithm

Calculating the development of spacecraft potentials in time is made difficult by the wide disparity between the time-scales for overall and differential charging. The basic equation of spacecraft charging is a multidimensional form of the equation for charging a capacitor:

$$C\, dV/dt = J(V)$$

It was immediately recognized that, due to the "stiffness" of the equations, the "explicit" form

$$C(V(t)-V(0)) = J(V(0))t$$

would be unstable (*i.e.*, the solution would oscillate unless very short timesteps were used). Accordingly, NASCAP/GEO uses the "implicit" form

$$C(V(t)-V(0)) = J(V(t))t$$

which requires the further approximation

$$J(V(t)) \approx J(V(0)) + \partial J/\partial V\ (V(t)-V(0))$$

(Estimating dJ/dV brings in still further complexity.) The implicit solution is

$$V(t)-V(0) = J(V(0))t/(C - t\, \partial J/\partial V)$$

which is stable as long as $\partial J/\partial V$ is negative.
The "method of patches" involves recognizing that the equation

$$C\, dV/dt = J(V(0)) + \partial J/\partial V\ (V-V(0))$$

has an exact solution

$$V - V(0) = -[J(V(0))/(\partial J/\partial V)][1-\exp(t\, (\partial J/\partial V)/C)]$$

which will give a better representation of the time-dependence of charging.

Figures 3 and 4 show calculations for charging of a single insulating surface on a grounded spacecraft using fairly long timesteps. The first figure shows the simpler case of charging in eclipse and the second the much more difficult case of discharging in sunlight. In both cases the explicit method gives clearly unphysical results. The implicit method works well in these cases, but underestimates the charging rate. The "method of patches" is a clear, albeit modest, improvement.

To apply the "method of patches" to a multiple-surface problem requires writing the charging equation in matrix

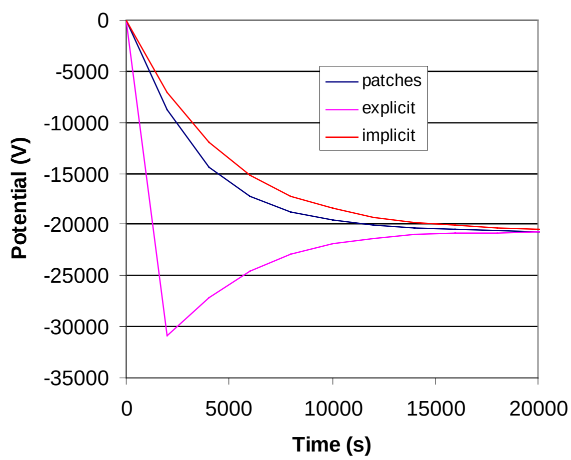

Figure 3. Results of the three time integration methods for charging a spacecraft insulating surface in eclipse.

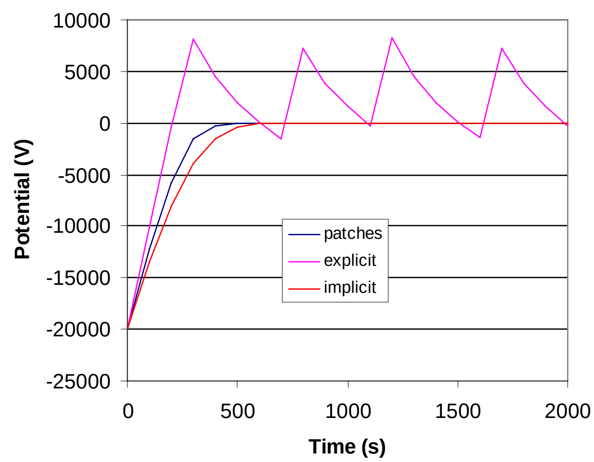

Figure 4. Results of the three time integration methods for discharging a spacecraft insulating surface in sunlight.

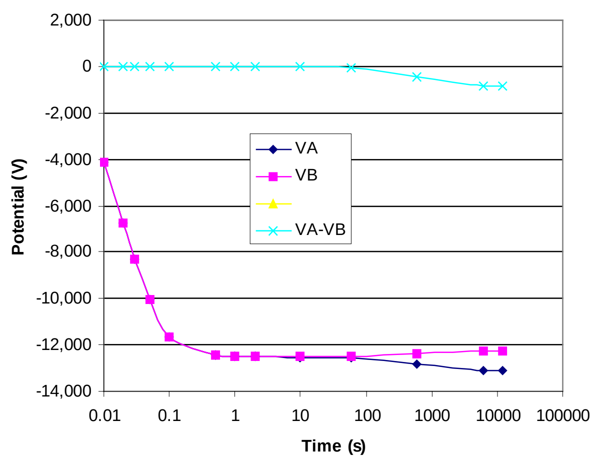

Figure 5. Charging of a 2-surface spacecraft with moderate timesteps.

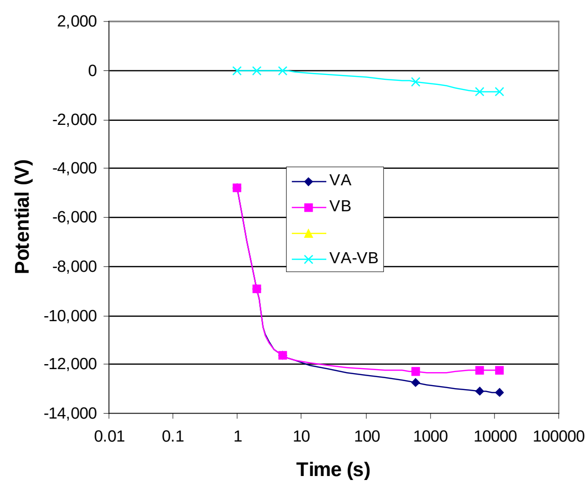

Figure 6. Charging of a 2-surface spacecraft with very long timesteps.

form and diagonalizing it. We start with the basic charging equation

$$\mathbf{C}\dot{V} = I(V)$$

where $\mathbf{C}$ is the capacitance matrix, and V and I are vectors. Linearizing gives

$$\mathbf{C}\dot{V} = A + \mathbf{B}V$$

If only plasma currents are considered, $\mathbf{B}$ may be approximated as diagonal. However, conductivity processes lead to off-diagonal elements in $\mathbf{B}$. To simultaneously diagonalize $\mathbf{B}$ and $\mathbf{C}$ we (1) apply a unitary transformation $\mathbf{M}_1$ to diagonalize $\mathbf{B}$ to $\mathbf{B}_1$; (2) apply a diagonal transformation $\mathbf{M}_2$ to transform $\mathbf{B}_1$ to the unit matrix; and (3) apply a final unitary matrix, $\mathbf{M}_3$, to diagonalize the already transformed capacitance matrix, $\mathbf{C}$. This gives

$$\mathbf{C}_3\dot{V}_3 = \mathbf{M_3 M_2 M}_1 A + V_3$$

where

$$\mathbf{C}_3 = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1\mathbf{C}\mathbf{M}_1^T\mathbf{M}_2\mathbf{M}_3^T$$
$$V_3 = \mathbf{M}_3\mathbf{M}_2^{-1}\mathbf{M}_1 V$$

Figures 5 and 6 show the application of the above method to a typical two surface spacecraft charging problem. In this problem, the eigenvalues of $\mathbf{C}_3$ are 764.5 and 0.048, quantifying the "stiffness" of the problem. Figure 5 uses timesteps that, although quite long, are adequate to resolve the charging time-scale. Figure 6 uses timesteps far too long toresolve the short time behavior, but nonetheless correctly integrates over the fast processes and correctly reproduces the slow processes.

Conclusions

We have discussed two methods for improving the calculation of spacecraft charging by NASCAP/GEO. Using the Boundary Element Method for surface potentials and electric fields will provide more accurate values for surface electric fields, while relaxing constraints on object definition. Using the “Method of Patches” for time integration will provide a more time-faithful and robust representation of dynamic charging.

These two methods are not independent of each other. For example, the external portion of the capacitance matrix needed for time-dependent charging is exactly the matrix generated by the BEM. The BEM can also be used to generate the relation of the time progress of electric field at one surface to the currents to other surfaces. The long-range goal would be to combine these two methods into a fully implicit method for advancing currents, potentials, and fields in a single process at each timestep.

References

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