Computation of Current to a Moving Bare Tether

Tatsuo Onishi*
Manuel Martinez-Sanchez†
Massachusetts Institute of Technology
Cambridge, Massachusetts

A Particle-In-Cell (PIC) method is developed and applied to simulate the electron current collection by a positively charged bare tether in both a quiescent unmagnetized and a flowing unmagnetized plasma conditions. In a quiescent case, the collected current calculated in our work is compared with the existing exact solution. The result is also plotted in a form suggested by theoretical work. Thus, the validity of the Orbital Motion Limit (OML) is also recognized in our particle simulation. Next, our work is extended to take account of the ion cross flow. As the wake region emerges, the assumption of a symmetric condition is violated, and the ions are no longer in Boltzmann equilibrium. Therefore the local density of ions depends on their mass, and so do the macroscopic quantities. Several simulations for the flowing plasma case are performed with different mass ratios. Although this case is oversimplified, some physical aspects of the plasma flow may be detected.

Introduction

The object of the work reported here is the development of a numerical method, PIC method, for the calculation of electron current to a positive bare tether moving at orbital velocity in the ionosphere. Current theory is limited to the case of a collisionless, quiescent, non-magnetized plasma, and to circular-cylinder geometry, except for the limiting case of (*size*) << (*Debye length*), called the OML, which applies to arbitrary cross-sectional shapes. On the other hand, the limits of the OML regime are only imperfectly known, as are the effects of wake formation due to cross-flow, magnetic presheath wings, and possible plasma instabilities. Accurate prediction of current collection is essential to the design and interpretation of data from the ProSEDS bare tether mission being designed by the NASA MSFC as well to potential future applications of electrodynamic tethers.

Preliminary results of our work on a PIC model for this purpose were presented in a post-deadline paper at the NASA Tether Technology Exchange Meeting in Sep. of 1997. Those results, limited to the symmetric, quiescent case, have been further extended and verified against the existing theory of Laframboise[2]. More recently, data from the quiescent case have been compared to theory in the form suggested by Sanmartin and Estes[3]. Also, the PIC model has been extended to the case with a plasma cross-flow of the order of the tether's orbital velocity (although with no magnetic field present). This extension has removed the symmetry restrictions, and has also forced the use of realistic electron/ion mass ratios, both of which increase the computation time substantially. Partially offsetting this, we have found that the presence of a supersonic plasma flow acts to stiffen the computation and accelerate its convergence. Preliminary indications are that the computed tether current is also less sensitive to the precise form of the boundary condition at the edge of the computational domain. Explorations are underway on the sensitivity of results to this and other numerical parameters (grid resolution, ion/electron mass ratio, domain size, number of superparticles, etc). Results will be presented in this paper indicating both, these sensitivities, and the calculated effect of cross of cross-flow and wake formation on the current.

Quiescent Unmagnetized Plasma

Electron current collection by the positive bare tether under a quiescent unmagnetized Maxwellian condition is calculated. In this case, due to the very high potential at the tether, the ion distribution may be assumed to be fully Maxwellian. On the other hand, since some electrons are absorbed by the tether, there is a region called "presheath" outside the sheath region. In the presheath region, quasineutrality prevails but the electric potential is not the same as at infinity. Because of the finite size of computational domain, we need to know the electric potential at the computational outside boundary. Given the potential, we can calculate the density due to inbound electrons and (both inbound and outbound) ions* , by integrating the Maxwell-Boltzmann distribution function in velocity space. The local density of ions

*Ph.D. Candidate
†Professor

*In this work, we call a particle which is coming in to the computational domain, an "inbound" particle, and one going out of the domain, an "outbound" particle.

may be given as follows;

$$n_i = n_\infty \exp(-\frac{|q|\phi}{kT_i}). \quad (1)$$

where n_i is ion density, n_∞ density at infinity, q electric charge of an electron, i.e. $-e$, ϕ potential, k Boltzmann constant and T_i ion temperature. The density of inbound electron may be given as

$$\begin{aligned} n_e^{in} &= \int_{-\infty}^{\infty}\int_{-\pi/2}^{\pi/2}\int_{\sqrt{|q|\phi/m_e}}^{\infty} f_e(\mathbf{v}) v dv d\theta dv_z \\ &= \frac{n_\infty}{2}, \end{aligned} \quad (2)$$

where $f_e(\mathbf{v})$ is the Maxwellian distribution function of electrons and ions respectively at potential ϕ.

$$f_e(\mathbf{v}) = n_\infty \left(\frac{m_e}{2\pi kT}\right)^{3/2} \exp\left(-\frac{\frac{1}{2}m_e(v^2+v_z^2)+q\phi}{kT}\right). \quad (3)$$

where m_e is the electron mass, v and v_z are velocities in the perpendicular and parallel directions to a tether, respectively. Applying these density to the quasineutral equation, we have

$$n_e^{out} + \frac{n_\infty}{2} = n_\infty \exp(-\frac{|q|\phi}{kT}). \quad (4)$$

Using the numerical method[1], the outbound electron density at the outside boundary n_e^{out} is obtained by

$$n_e^{out} = \frac{k_{out}^2}{\sum_{i=1}^{k} \mathbf{w}_{ei}\mathbf{n}_i dtS}. \quad (5)$$

where k_{out} is the number of electrons counted as they cross the boundary, $\mathbf{n}_i$ the normal vector to the boundary and $\mathbf{w}_{ei}$ the particle velocity. Also dt is the timestep and S is the area of the outside boundary. Then equation (4) gives the potential at the computational outside boundary.

Now we are ready to calculate how many electrons and ions are to be replenished at each timestep, namely the total flow rates of "inbound" partilces. The number of those particles is calculated by multiplying the flux by the timestep and the area of the outside boundary. The number of electrons to be replenished, k_e, is given

$$\begin{aligned} k_e &= \int_{-\infty}^{\infty}\int_{-\pi/2}^{\pi/2}\int_{\sqrt{|q|\phi/m_e}}^{\infty} f_e(\mathbf{v}) v^2 \cos\theta dv d\theta dv_z S dt \\ &= \frac{n_\infty \bar{c}}{4}\left\{\frac{2}{\sqrt{\pi}}\sqrt{\frac{|q|\phi}{kT}} + \exp\left(\frac{|q|\phi}{kT}\right) erfc\sqrt{\frac{|q|\phi}{kT}}\right\} S dt \end{aligned} \quad (6)$$

where $f_e(\mathbf{v})$ is given Eqn(3) and $\bar{c}$ is the random thermal velocity. And the number of ions to be replenished, k_i, is given

$$\begin{aligned} k_i &= \int_{-\infty}^{\infty}\int_{-\pi/2}^{\pi/2}\int_{0}^{\infty} f_i(\mathbf{v}) v^2 \cos\theta dv d\theta dv_z S dt \\ &= n_\infty \left(\frac{kT}{2\pi m_i}\right)^{1/2} \exp\left(-\frac{|q|\phi}{kT}\right) S dt, \end{aligned} \quad (7)$$

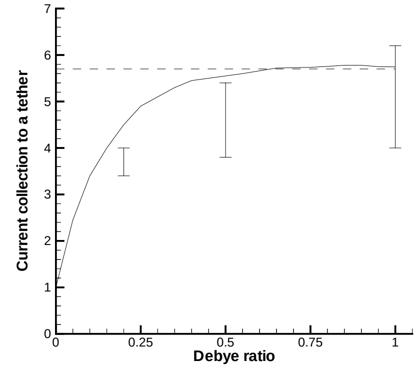

Figure 1: Current collection: OML current (dotted line), exact solution (solid line) and PIC calculation (error bar)

where $f_i(\mathbf{v})$ is the Maxwellian distribution function for ions. In the simulation, a random number generator is used to locate the injection place of the particle to be replenished and provide the particle with a velocity estimated from the Maxwellian distribution function.

Results

In this section, we show the results from the simulations. In order to validate our code in and near the OML regime, we performed simulations with three different parameters, i.e. *Debye ratio* $= 1.0$, 0.5 *and* 0.2. *Debye ratio* is defined as the ratio of the Debye length of the plasma to the radius of a tether. Other parameters used in those simulations are temperature ratio $T_i/T_e = 1$, potential ratio, $\psi_p = e\phi_p/kT_i = 25$, and mass ratio, $m_i/m_e = 1$. This non-physical mass ratio is verified to speed up the computation without loss of quality in the result[1], basically because of the ion equilibrium state (Fig. 1).

The results thus obtained are compared with the existing exact solution [2]. In Figure 2, electric charge density and potential distributions and the current collection versus time are shown. Electric charge density and potential are normalized by en_∞ and kT_i/e, respectively, and the current collection is normalized by the random thermal current given by

$$I = Sn_\infty|q|\sqrt{\frac{kT_{i,e}}{2\pi m_{i,e}}} \quad (8)$$

For comparison with the exact solution the current collection is also shown in Figure 1. The "exact" solution is taken from ref.[5].

Next, the potential distribution is plotted versus $1/r^2$ as suggested by Sanmartin and Estes (Figure 3). The potential profile at the top corresponds to the case of Debye ratio equal to unity, which is on the boundary of the

OML regime. As *Sanmartin and Estes*[3] discuss, when the Debye ratio decreases, the potential profile shifts down into the region where the OML current is not obtained. When the current collection is approximated by the OML, the potential profile is on and/or above the boundary, indicated by the dashed line in Figure 3. In the case of Debye ratio = 1, collected current is near the limit of the OML current, and accordingly the corresponding potential profile touches the boundary but never get below it.

In conclusion, the comparison with the exact solution shows that our code provides good qualitative and quantitative approximations.

Flowing Unmagnetized Plasma

Next, the flowing unmagnetized case is considered. A few computations are performed in a regime which would be the OML regime in a quiescent case, with different mass ratios, giving rise to different wake configurations. Because of the emergence of the wake, and the nonzero motion at infinity, the electron and ion distribution functions are not analytically known once the potential is disturbed. For the present computations, we simpy used $n_e \bar{c}_e/4$ for the in bound electron flux, and for ions we took the inward normal component of the flux at infinity, $n_e U_{tether}$. Also the potential at the computational outside boundary is obtained by the zero-gradient condition at the boundary. Electric charge density and potential distributions are shown in Figures 4 and 5, respectively.

In this flowing case, the speed of a tether was chosen to satisfy the meso-thermal condition,

$$v_{te} >> U_{tether} >> v_{ti}$$

, where $v_{te,i}$ are electron and ion thermal velocities and U_{tether} is the orbital speed of a tether. For mass ratio 10000 and 100, U_{tether}'s are given as

$$U_{tether} = 8 \times v_{ti}$$

and

$$U_{tether} = 5 \times v_{ti}$$

respectively. From figure 4, the effect of the ion inertia on the wake configuration can be noticed. Since ions come closer to the tether in the case of mass ratio 10000 than 100, thus the size of sheath is smaller (Fig. 5). Due to the zero-gradient condition on the outside boundary, the potential at the boundary takes non-physical negative values, requiring a shift of the ϕ scale. Consequently the fact that no ions are absorbed by the tether is shown by comparing the potential at the tether to the ion's kinetic energy as follows,

$$\frac{e\phi_p}{\frac{1}{2}m_i U_{tether}^2} = \frac{2\psi_p}{(U_{tether}/v_{ti})^2} = 2 \ (for\ mass\ ratio = 10000)$$
$$= 3.52 \ (for\ mass\ ratio = 100)$$

Although this case is oversimplified, there are some interesting physics to be noticed. As seen in Figure 5, the normalized potential at the tether with respect to infinity is practically $\psi_p = 64$ and 44 for the cases of mass ratio 10000 and 100 respectively. Electrons are injected in to the computational domain, as if at the local boundary point, the potential were zero. Therefore as far as the movement of electrons is concerned, the only difference from the quiescent case is the potential field, i.e. the value of the potential at the tether and, to some extent, the presence of a wake region. Figure 6 shows the current collection in those cases. These currents are also normalized by the random thermal current. Considering the fact that in the OML regime, the current collected by a tether is proportional to the square root of the potential at the tether, we can (Table 1) see that in both cases of mass ratio, 10000 and 100, currents are still roughly in the OML regime. Therefore we can tentatively conclude that in the OML regime the wake configuration in the potential profile does not affect the current collection into a tether.

Acknowledgment

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References

[1] Onishi,T. and Martinez-Sanchez, M., Electron Current Collection By a Bare Wire Using Particle-In-Cell Method, *AIAA paper 98-2828*, June 1998.

[2] Laframboise, J. G., Theory of spherical and cylindrical langmuir probe in a collisionless, maxwellian plasma at rest. Technical Report 100, University of Toronto, Institute of Aerospace Studies Report, 1966.

[3] Sanmartin, J.R. and Estes, R.D., The Orbital Motion Limited Regime of Cylindrical Langmuir Probes. submitted for publication to the *Journal of Geophysical Research*, 1998.

[4] Goldstein, H. *Classical Mehnics*. Addison-Wesley Pub. Co., Massachusetts, 1980.

[5] Chung P. M., Talbot L. and Touryan K. J.,*Electric Probes in Stationary and Flowing Plasmas; Theory and Application*. Springer-Verlag,1975.

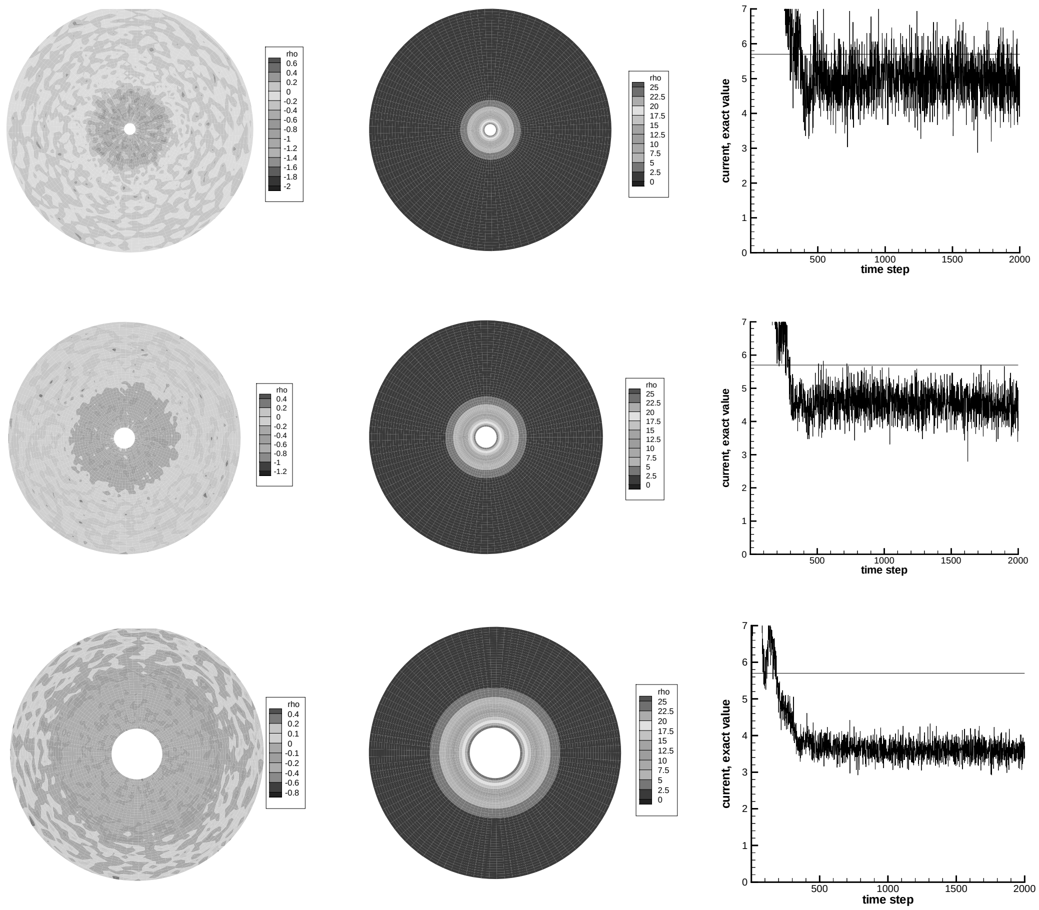

Figure 2: Quiescent unmagnetized plasma: (from left to right) Electric c harge density, potential and current collection vs. time step. (From top to bottom) Debye ratio = 1.0, 0.5 and 0.2

case	*Tether Potential*	*Collected Current*	$Current/\sqrt{Potential}$
No ion motion (Debye ratio = 1.0, Fig. 1)	$\psi_p = 25$	5.2	10.4
Ion motion, $m_i/m_e = 10000$	$\psi_p = 64$	8.8	10.9
Ion motion, $m_i/m_e = 100$	$\psi_p = 44$	7.0	10.7

Table 1: Ratio of current to the square root of potential, ψ is the normalized potential

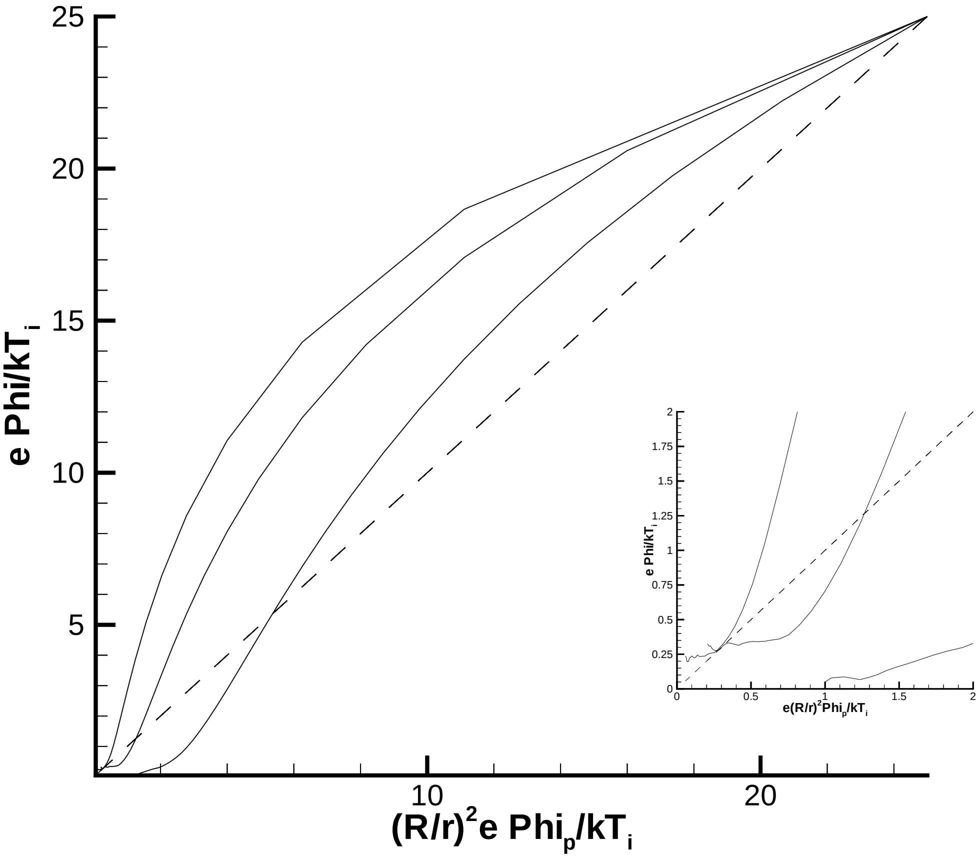

Figure 3: P oten tial profile and the boundary of the OML regime (From top to bottom) Debye ratio = 1.0, 0.5 and 0.2

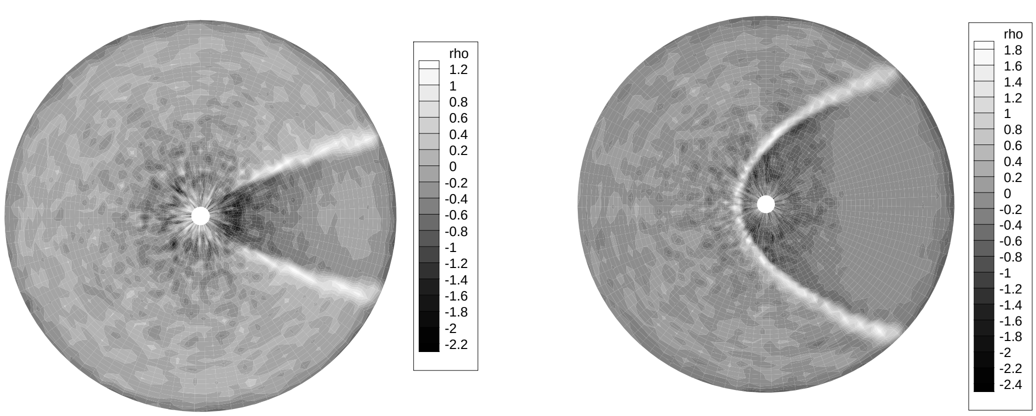

Figure 4: Electric c harge density in the flowing unmagnetized case : (Left) Mass ratio = 10000 and (Right) Mass ratio = 100

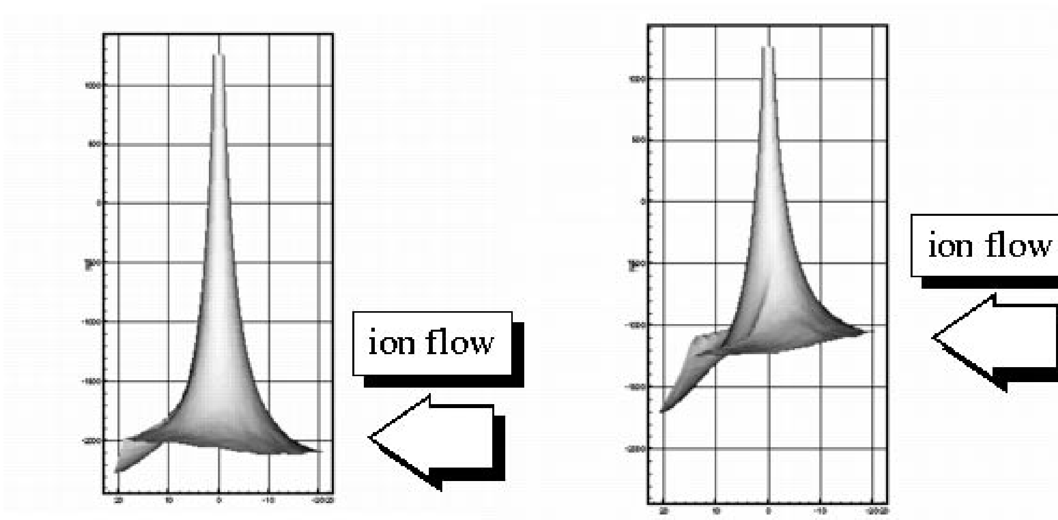

Figure 5: Electric potential (side view) in the flowing unmagnetized case : (Left) Mass ratio = 10000 and (Right) Mass ratio = 100. The potential in this figure is not normalized, In both cases, potential is normalized by the factor $kT_i/e = 50$.

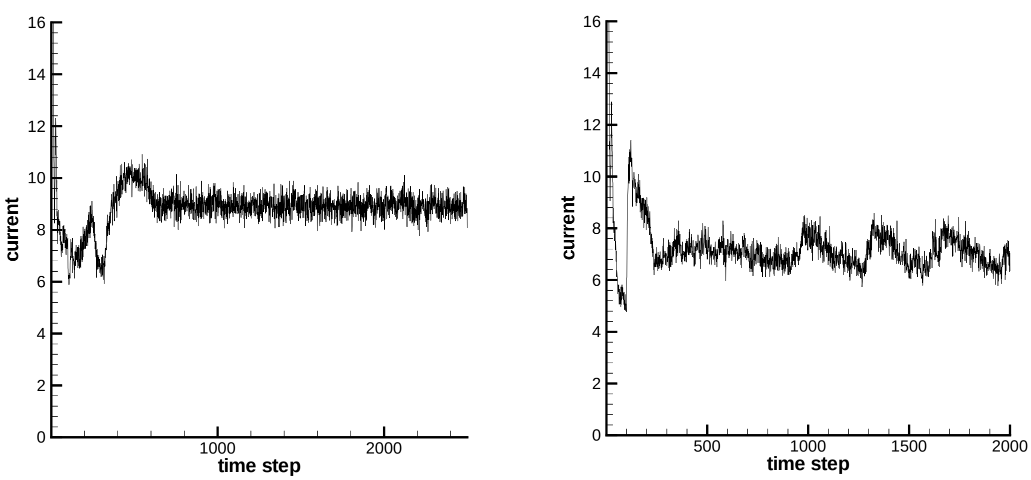

Figure 6: Current collection in the flowing unmagnetized case : (Left) Mass ratio = 10000 and (Right) Mass ratio = 100